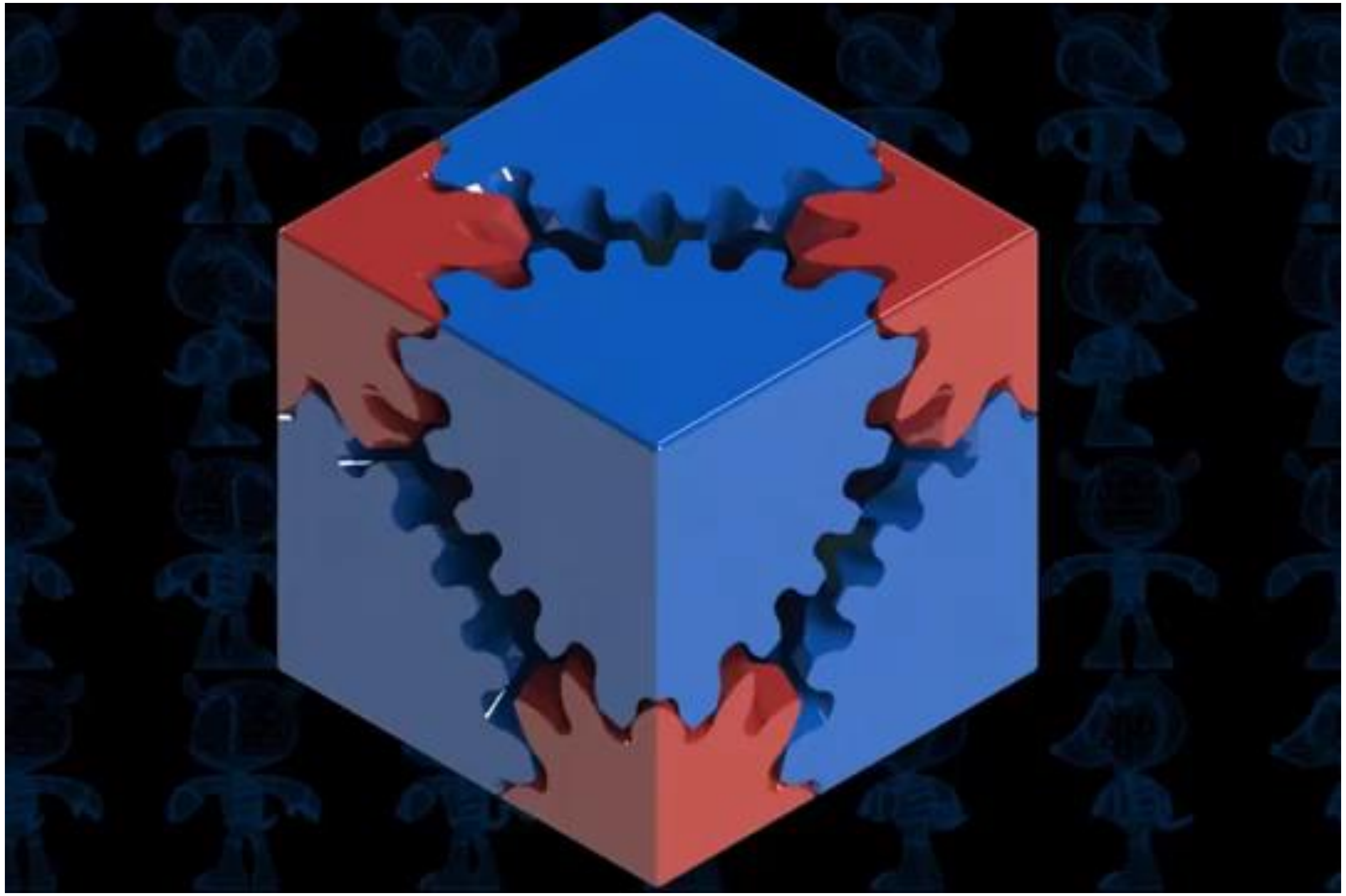
Kinematics of Mechanisms (Simulation & Design)

Mechanisms and kinematics

Examples of mechanisms

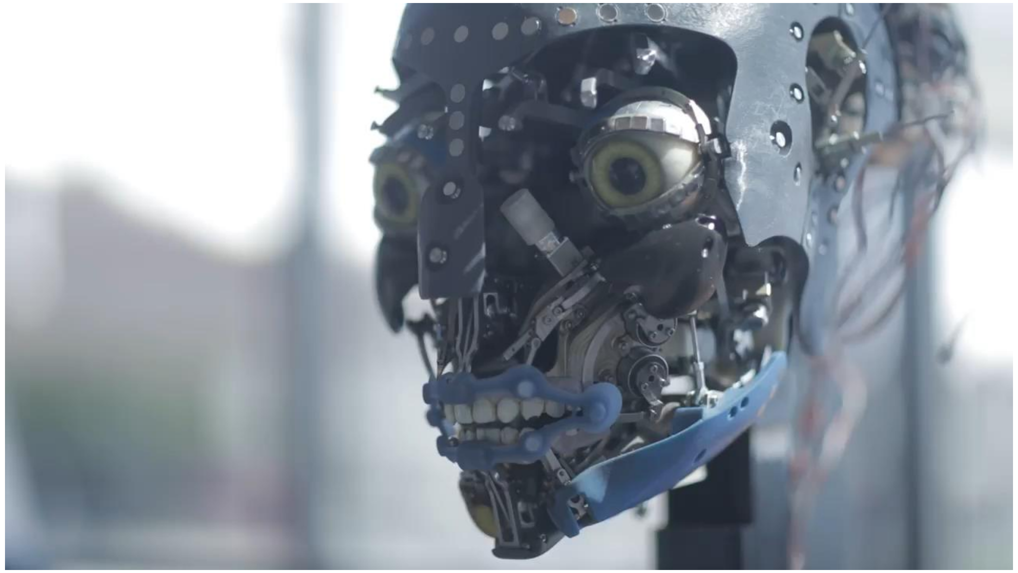
With different components such as linkages, cams, gears etc.

https://www.youtube.com/watch?v=7YegXe_S1ys&feature=youtu.be



Mechanisms and kinematics

Artistic expression using mechanisms



https://www.youtube.com/watch?v=bFU9Qg_6EsY

https://www.youtube.com/watch?v=uTTezk__Xvw

Mechanisms and kinematics

An example in healthcare industry: prosthetics

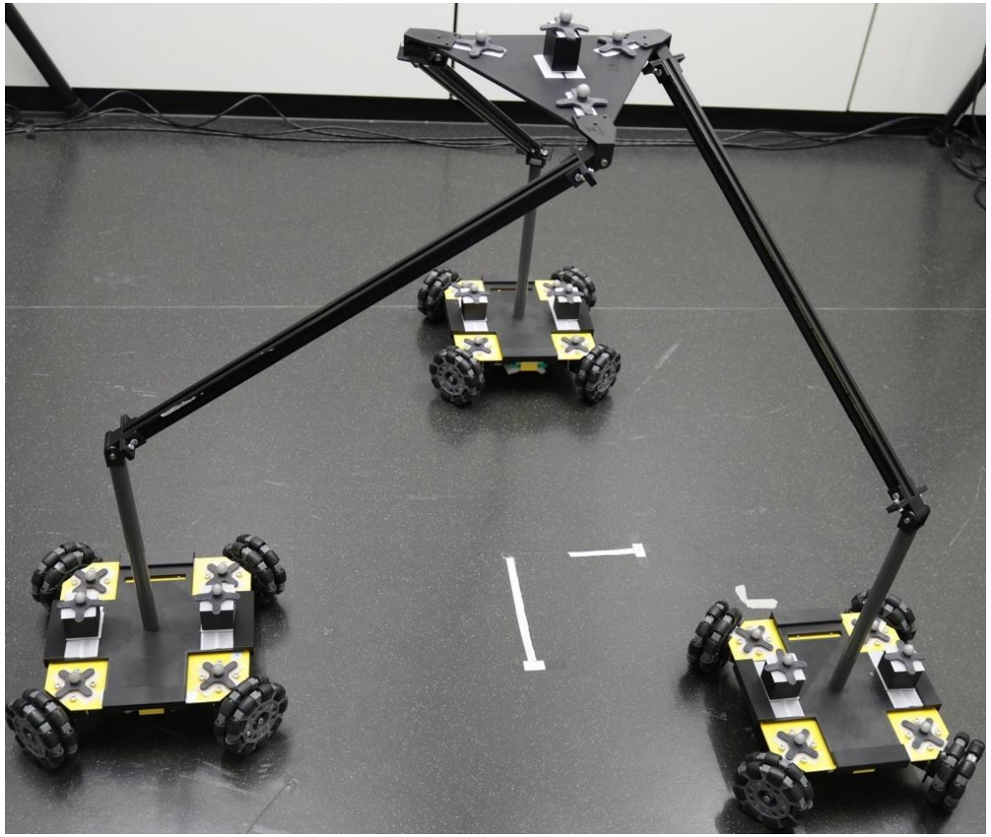
https://www.facebook.com/TheTechViral/videos/308959139761924/

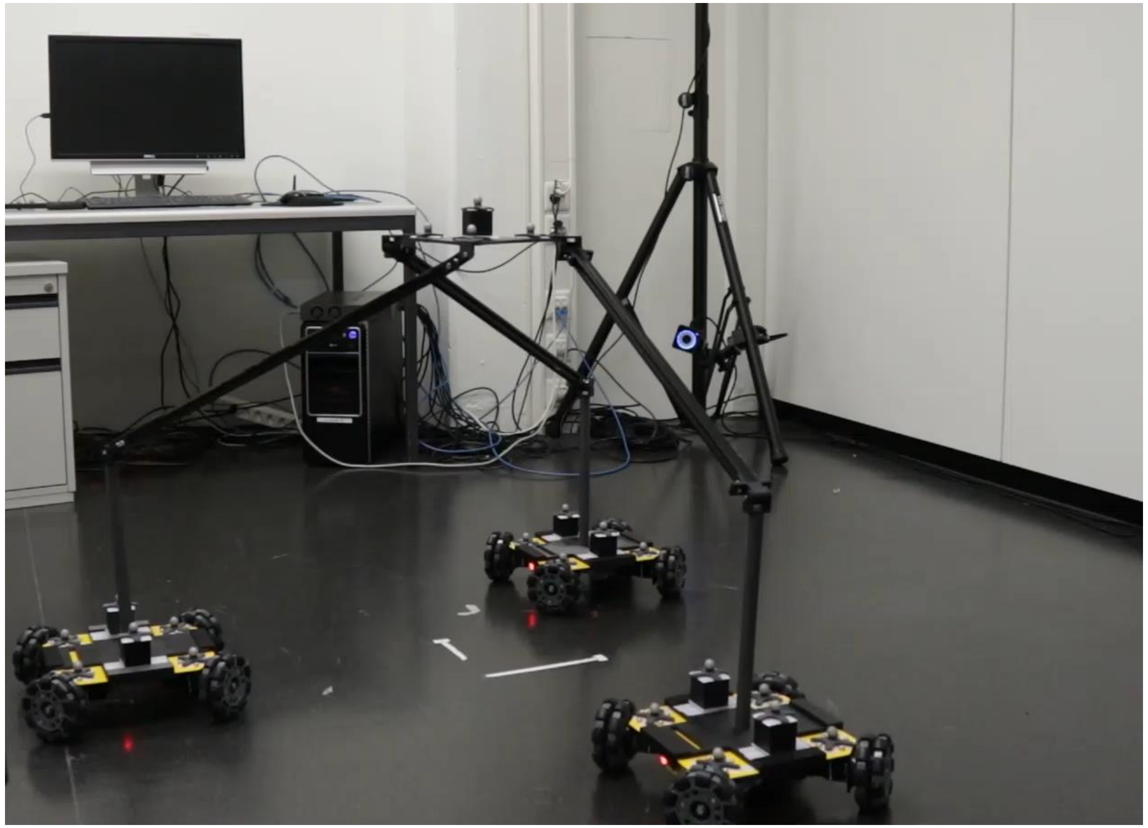
Mechanisms and kinematics

Industrial manipulators: motion control and automation

Mechanisms and kinematics

Collaborative manipulation of a mechanism with multiple mobile robots



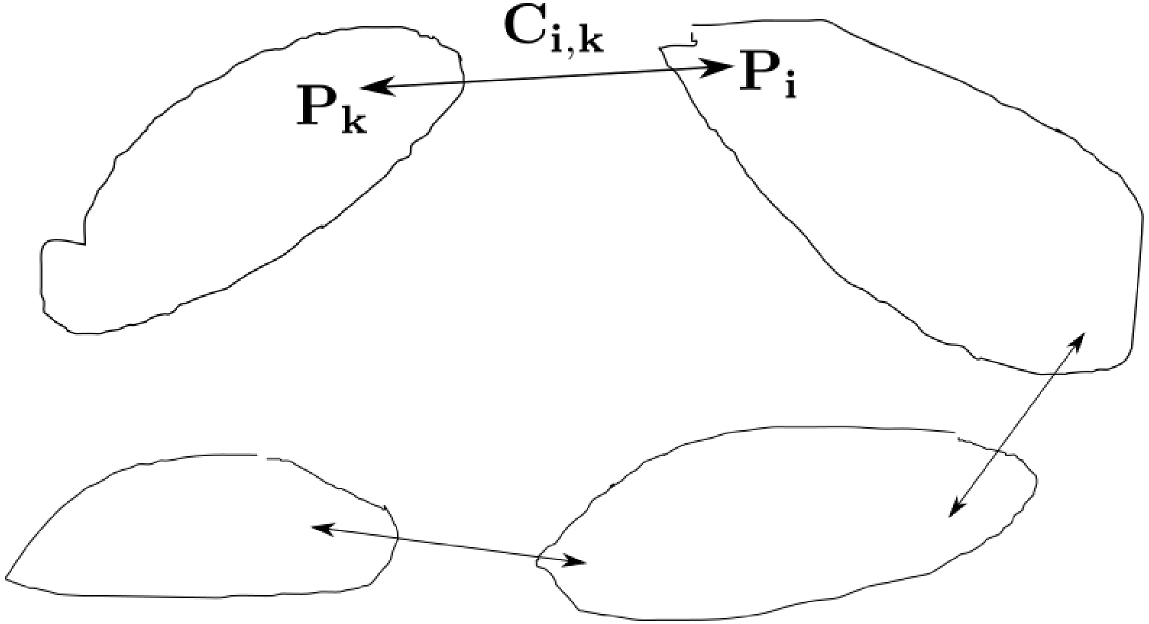


https://www.dropbox.com/s/nh1ru80p2u9avoo/CCMA.mp4?dl=0
(download video here)

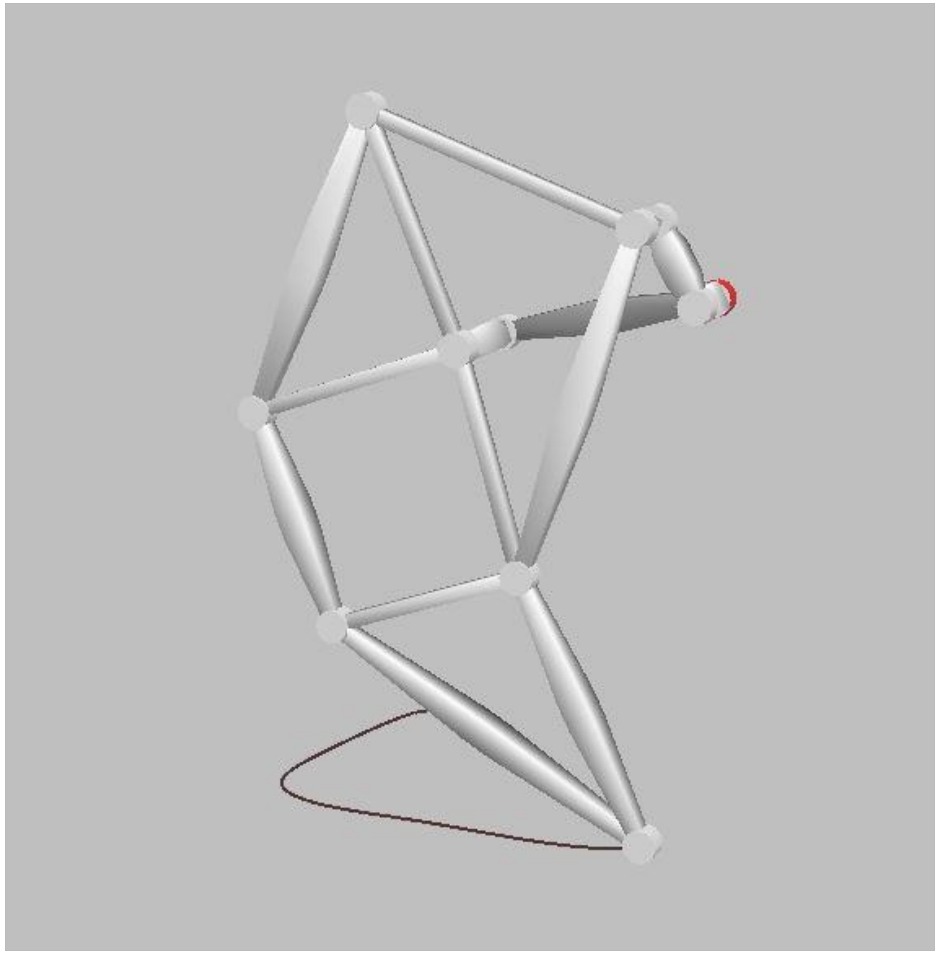
“What is a mechanism”

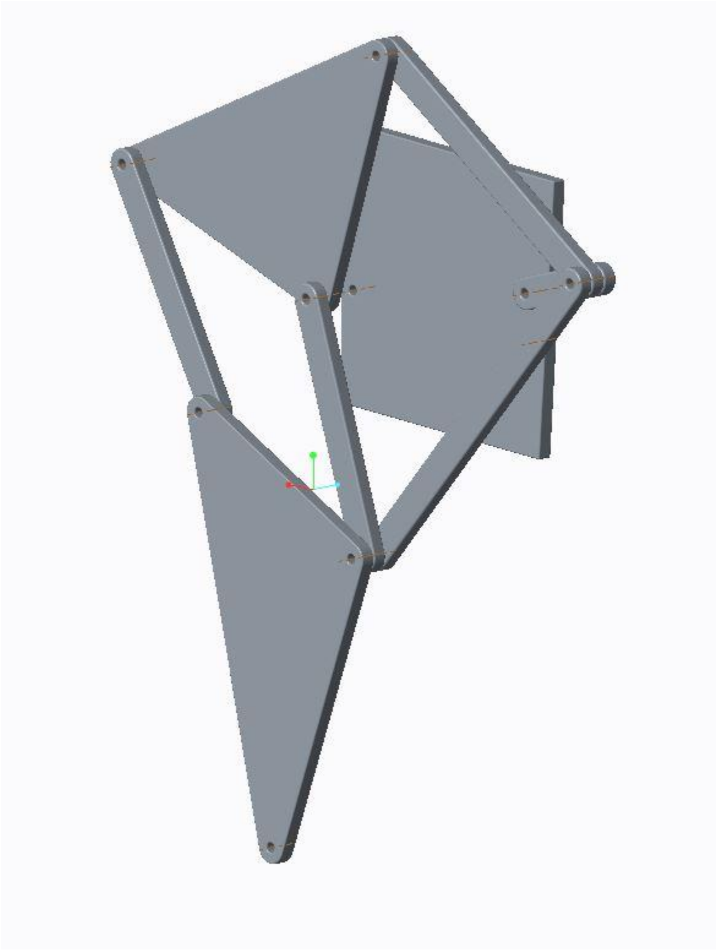
- Collection of “**rigid bodies**” “**connected**” with “**constraints**”
- Rigid bodies are indexed 0,1..n-1 with $\mathbf{s_i}$ stacked in a vector

    $\mathbf{s}$: mechanism state vector with 3*n DOF



Theo Jansen linkage mechanism

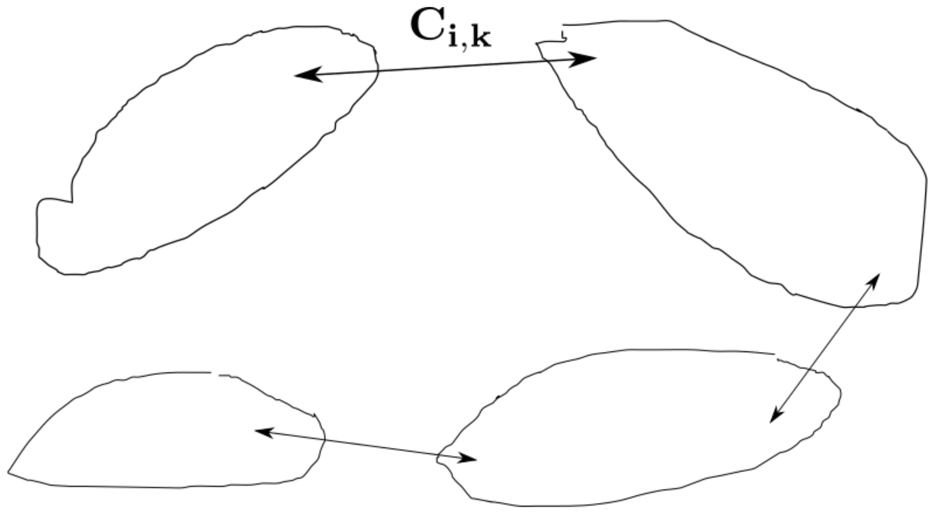




Simulation of a mechanism

- Collection of "**rigid bodies**" "**connected**" with "**constraints**"
- From each **connection** between two rigid bodies with index i and k, collect all **constraints** in a single vector of constraints $\mathbf{C}$
- Determine state $\mathbf{s}$ of the mechanism to satisfy all constraints

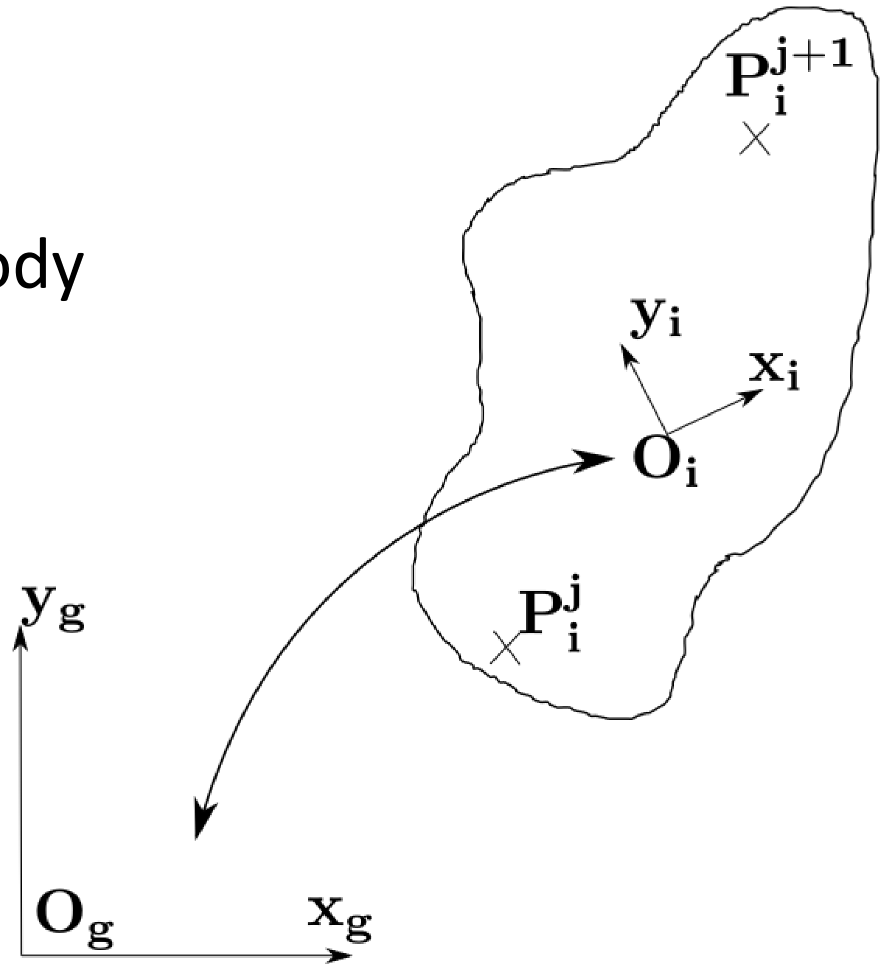
$$\hat{\mathbf{s}} = \min_{\mathbf{s}} \; E(\mathbf{s}) = \frac{1}{2}\mathbf{C}^T\mathbf{C}$$



World and local reference frame

- The global reference frame at $\mathbf{O_g}$
- A local reference frame at $\mathbf{O_i}$
- Points $\mathbf{P_i^j}$ define the shape of the rigid body
- Planar rigid body - 3DOF
- State $\mathbf{s_i}$ of the rigid body

$$\mathbf{s_i} = [\theta_i \quad \mathbf{O_i}[0] \quad \mathbf{O_i}[1]]$$



World and local reference frame

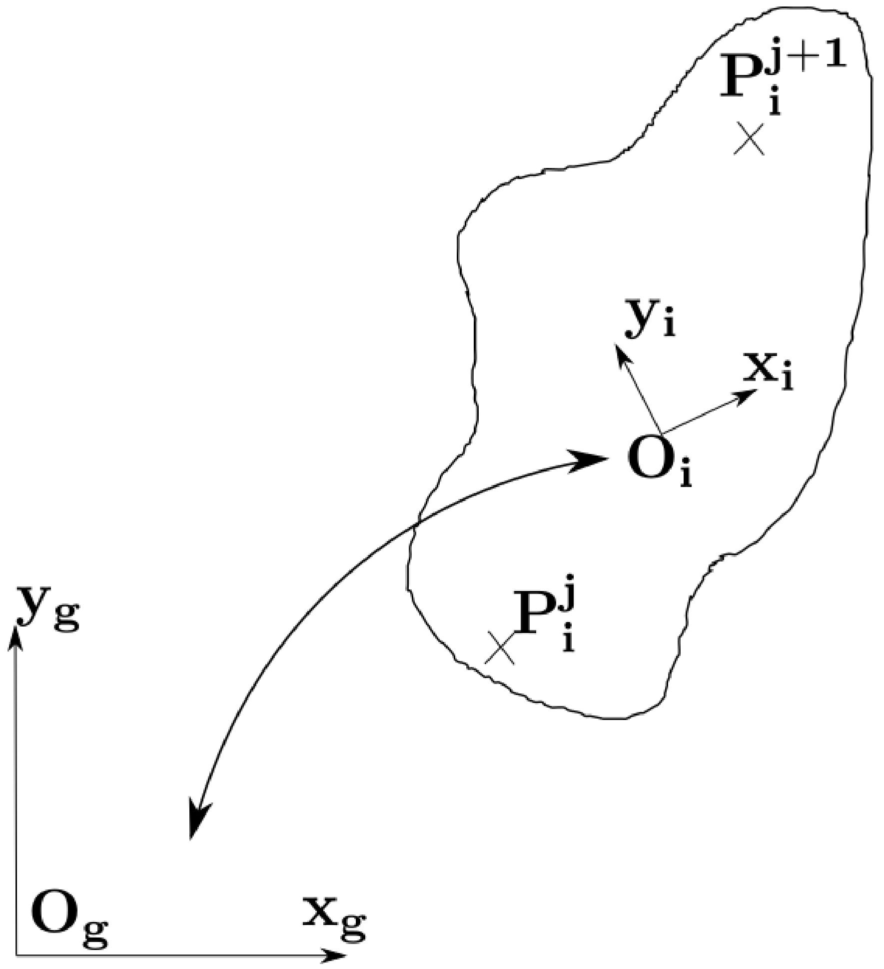
- $\mathbf{s_i} = [\theta_i \quad \mathbf{O_i}[0] \quad \mathbf{O_i}[1]]$

- Expressing points from local to global reference frame

$$\mathbf{P_i^j} = \mathbf{R}(\theta_\mathbf{i}) \cdot \bar{\mathbf{P}}_\mathbf{i}^\mathbf{j} + \mathbf{O_i}$$

- $\mathbf{R}$ is a rotation matrix;

$$\mathbf{R}^T \cdot \mathbf{R} = \mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}$$



Modeling

- Collection of "**rigid bodies**" "**connected**" with "**constraints**"
- **Rigid bodies** are indexed 0,1..n-1 with $\mathbf{s_i}$ stacked in a vector

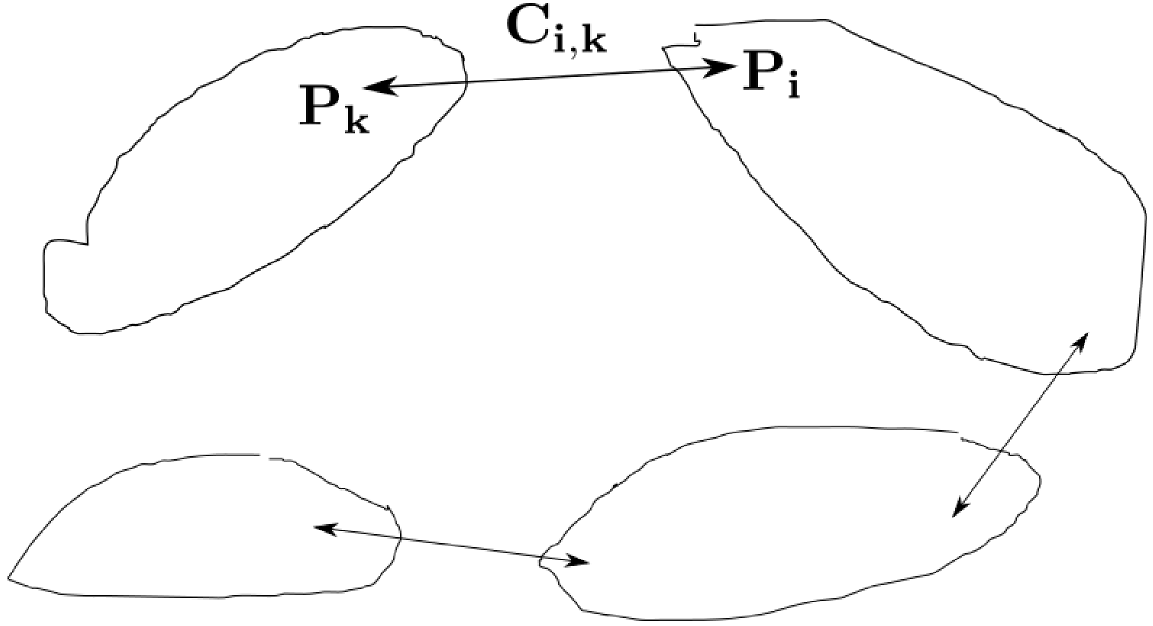
  $\mathbf{s}$: mechanism state vector with 3*n DOF

- Some common **constraints** are:
  - ground component constraint

$$\mathbf{s_i} - \mathbf{s_i}^* = \mathbf{0}$$

  - point on point constraint

$$\mathbf{P_i} - \mathbf{P_k} = \mathbf{0}$$



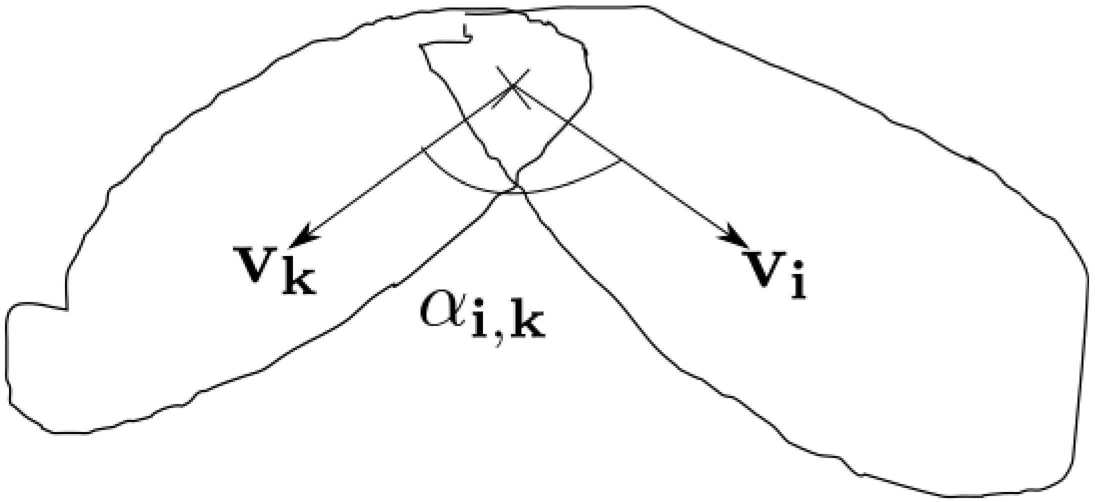
Modeling

- Collection of "**rigid bodies**" "**connected**" with "**constraints**"
- **Rigid bodies** are indexed 0,1..n-1 with $\mathbf{s_i}$ stacked in a vector

  $\mathbf{s}$: mechanism state vector with 3*n DOF
- Some common **constraints** are:
  - Vector on Vector constraint
  - $\alpha_{\mathbf{i,k}}$: motor angle

$$\mathbf{P_i} - \mathbf{P_k} = \mathbf{0}$$

$$\mathbf{v_k} - \mathbf{R}(\alpha_{i,k}) \cdot \mathbf{v_i} = \mathbf{0}$$



Modeling

- Collection of "**rigid bodies**" "**connected**" with "**constraints**"
- **Rigid bodies** are indexed 0,1..n-1 with $\mathbf{s_i}$ stacked in a vector

  $\mathbf{s}$: mechanism state vector with 3*n DOF
- Some common **connections** with **corresponding constraints** are:
  - Ground connection $$\mathbf{s_i} - \mathbf{s_i}^* = \mathbf{0}$$
  - Revolute joint connection $$\mathbf{P_i} - \mathbf{P_k} = \mathbf{0}$$
  - Servo motor connection (actuator connection) $$\mathbf{v_k} - \mathbf{R}(\alpha_{i,k}) \cdot \mathbf{v_i} = \mathbf{0}$$ $$\mathbf{P_i} - \mathbf{P_k} = \mathbf{0}$$

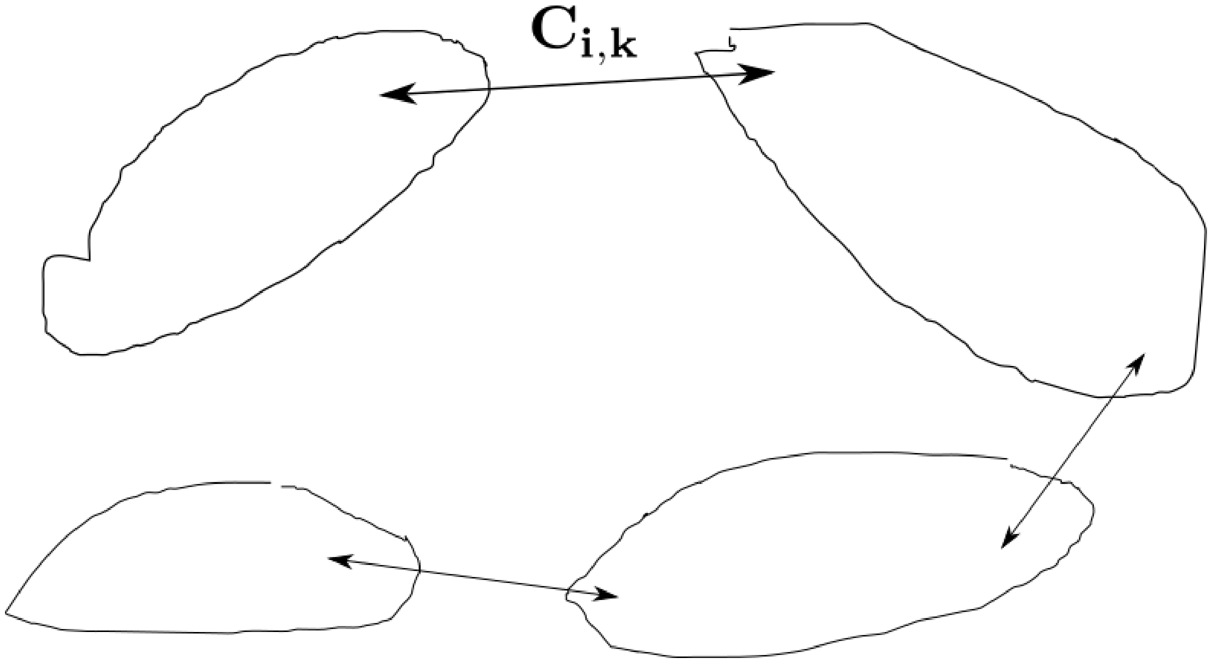
Simulation of the mechanism

From each connection between two rigid bodies with index i and k, collect all constraints in a single vector of constraints $\mathbf{C}$

$$\mathbf{s_i} - \mathbf{s_i}^* = \mathbf{0}$$

$$\mathbf{P_i} - \mathbf{P_k} = \mathbf{0}$$

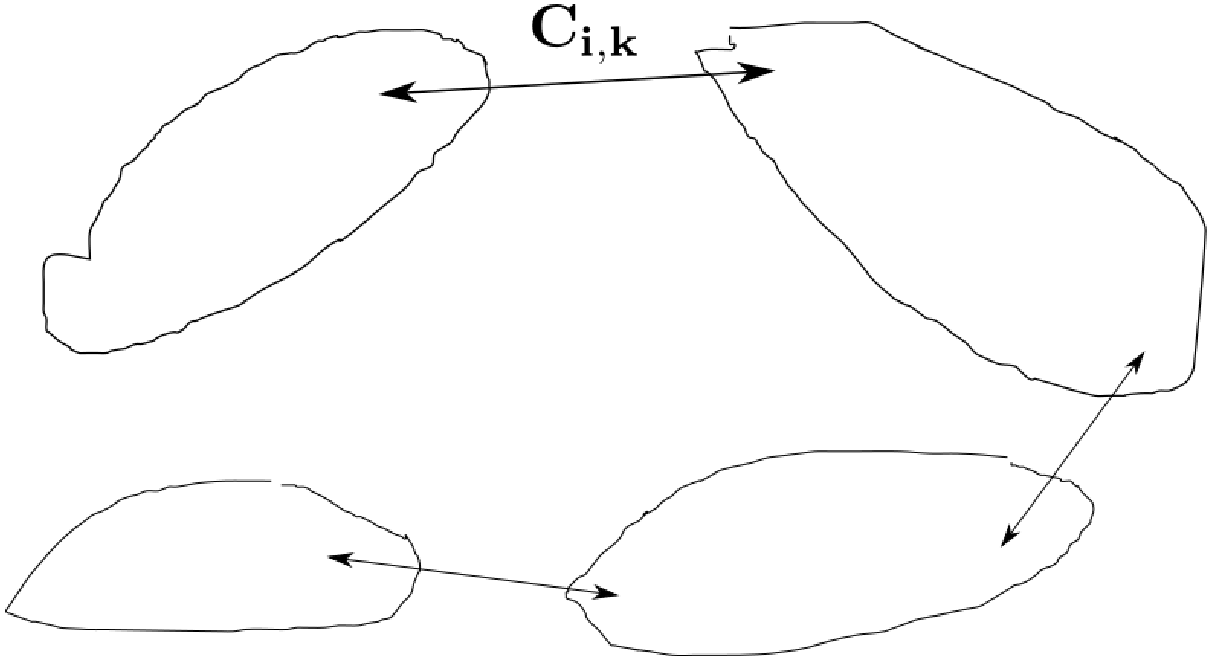
$$\mathbf{v_k} - \mathbf{R}(\alpha_{i,k}) \cdot \mathbf{v_i} = \mathbf{0}$$



Simulation of the mechanism

From each connection between two rigid bodies with index i and k, collect all constraints in a single vector of constraints $\mathbf{C}$

$$\hat{\mathbf{s}} = \min_{\mathbf{s}} E(\mathbf{s}) = \frac{1}{2}\mathbf{C}^T\mathbf{C}$$



Need to calculate derivatives

$$\frac{dE}{d\mathbf{s}} = \frac{d\mathbf{C}}{d\mathbf{s}}^T\mathbf{C}$$

Simulation of the mechanism

From each connection between two rigid bodies with index i and k, collect all constraints in a single vector of constraints $\mathbf{C}$

Need to calculate derivatives

$$\mathbf{s_i} - \mathbf{s_i}^* = \mathbf{0}$$

$$\mathbf{P_i} - \mathbf{P_k} = 0$$

$$\mathbf{v_k} - \mathbf{R}(\alpha_{i,k}) \cdot \mathbf{v_i} = \mathbf{0}$$

$$\frac{dE}{d\mathbf{s}} = \frac{d\mathbf{C}}{d\mathbf{s}}^T \mathbf{C}$$

What is $\frac{d\mathbf{P_i}}{d\mathbf{s}}$ for example:

Simulation of the mechanism

From each connection between two rigid bodies with index i and k, collect all constraints in a single vector of constraints $\mathbf{C}$

Need to calculate derivatives $\frac{dE}{d\mathbf{s}} = \frac{d\mathbf{C}}{d\mathbf{s}}^T \mathbf{C}$

What is $\frac{d\mathbf{P_i}}{d\mathbf{s}}$ for example:

$$\mathbf{s_i} = [\theta_i \quad \mathbf{O_i}[0] \quad \mathbf{O_i}[1]] \qquad \mathbf{P_i^j} = \mathbf{R}(\theta_\mathbf{i}) \cdot \bar{\mathbf{P}}_\mathbf{i}^\mathbf{j} + \mathbf{O_i}$$

$$\frac{d\mathbf{P_i}}{d\mathbf{O}_i} = \mathbf{I} \qquad \frac{d\mathbf{P_i}}{d\theta_i} = \frac{d\mathbf{R}(\theta_\mathbf{i})}{d\theta_i} \cdot \bar{\mathbf{P}}_\mathbf{i} \qquad \frac{d\mathbf{R}(\theta_\mathbf{i})}{d\theta_i} = \mathbf{W}_\times \cdot \mathbf{R}$$

Proof 1

$$\mathbf{R}^T \cdot \mathbf{R} = \mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}$$

$$\dot{\mathbf{R}} \cdot \mathbf{R}^T + \mathbf{R} \cdot \dot{\mathbf{R}}^T = \mathbf{0}$$

$$\dot{\mathbf{R}} \cdot \mathbf{R}^T = -\mathbf{R} \cdot \dot{\mathbf{R}}^T = \mathbf{W}_\times$$

$$\dot{\mathbf{R}} \cdot \mathbf{R}^T \mathbf{R} = \mathbf{W}_\times \cdot \mathbf{R}$$

$$\dot{\mathbf{R}} \cdot \mathbf{I} = \mathbf{W}_\times \cdot \mathbf{R}$$

$$\dot{\mathbf{R}} = \mathbf{W}_\times \cdot \mathbf{R}$$

Simulation of the mechanism

$$\mathbf{s_i} = [\theta_i \quad \mathbf{O_i}[0] \quad \mathbf{O_i}[1]] \qquad \mathbf{P_i^j} = \mathbf{R}(\theta_\mathbf{i}) \cdot \bar{\mathbf{P}}_\mathbf{i}^\mathbf{j} + \mathbf{O_i}$$

$$\frac{d\mathbf{P_i}}{d\mathbf{O}_i} = \mathbf{I} \qquad \frac{d\mathbf{P_i}}{d\theta_i} = \frac{d\mathbf{R}(\theta_\mathbf{i})}{d\theta_i} \cdot \bar{\mathbf{P}}_\mathbf{i} \qquad \frac{d\mathbf{R}(\theta_\mathbf{i})}{d\theta_i} = \mathbf{W}_\times \cdot \mathbf{R}$$

- **What is $\mathbf{W}_\times$ and its physical meaning?**
- Each rotation matrix $\mathbf{R}$ is associated with a unit rotation vector $\mathbf{W}$

  $\mathbf{W}_\times$ is the skew symmetric matrix corresponding to this vector $\mathbf{W}$

Proof 2

- We have already seen this

$$\mathbf{P_i^j} = \mathbf{R}(\theta_\mathbf{i}) \cdot \mathbf{\bar{P}_i^j} + \mathbf{O_i}$$

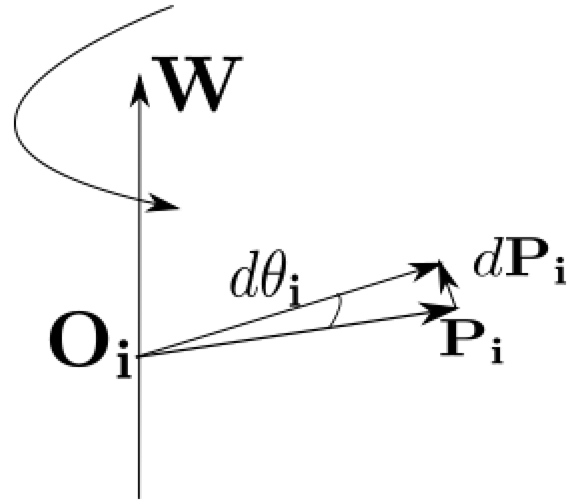
$$\frac{d\mathbf{P_i}}{d\theta_i} = \frac{d\mathbf{R}(\theta_\mathbf{i})}{d\theta_i} \cdot \mathbf{\bar{P}_i}$$

- For a unit vector $\mathbf{W}$ representing axis of rotation passing through $\mathbf{O_i}$

$$\frac{d\mathbf{P_i}}{d\theta_i} = \mathbf{W}_\times \cdot (\mathbf{P_i} - \mathbf{O_i})$$

$$\frac{d\mathbf{P_i}}{d\theta_i} = \mathbf{W}_\times \cdot \mathbf{R}(\theta_\mathbf{i}) \cdot \mathbf{\bar{P}_i}$$

thus $\mathbf{W}_\times \cdot \mathbf{R}(\theta_\mathbf{i}) = \frac{d\mathbf{R}(\theta_i)}{d\theta_i}$



Interactively editing the design of the linkage based mechanism changes the output motion

https://www.dropbox.com/s/l0py7dlq0z08kjp/theojansen_Trim.mp4?dl=0 (download video here)