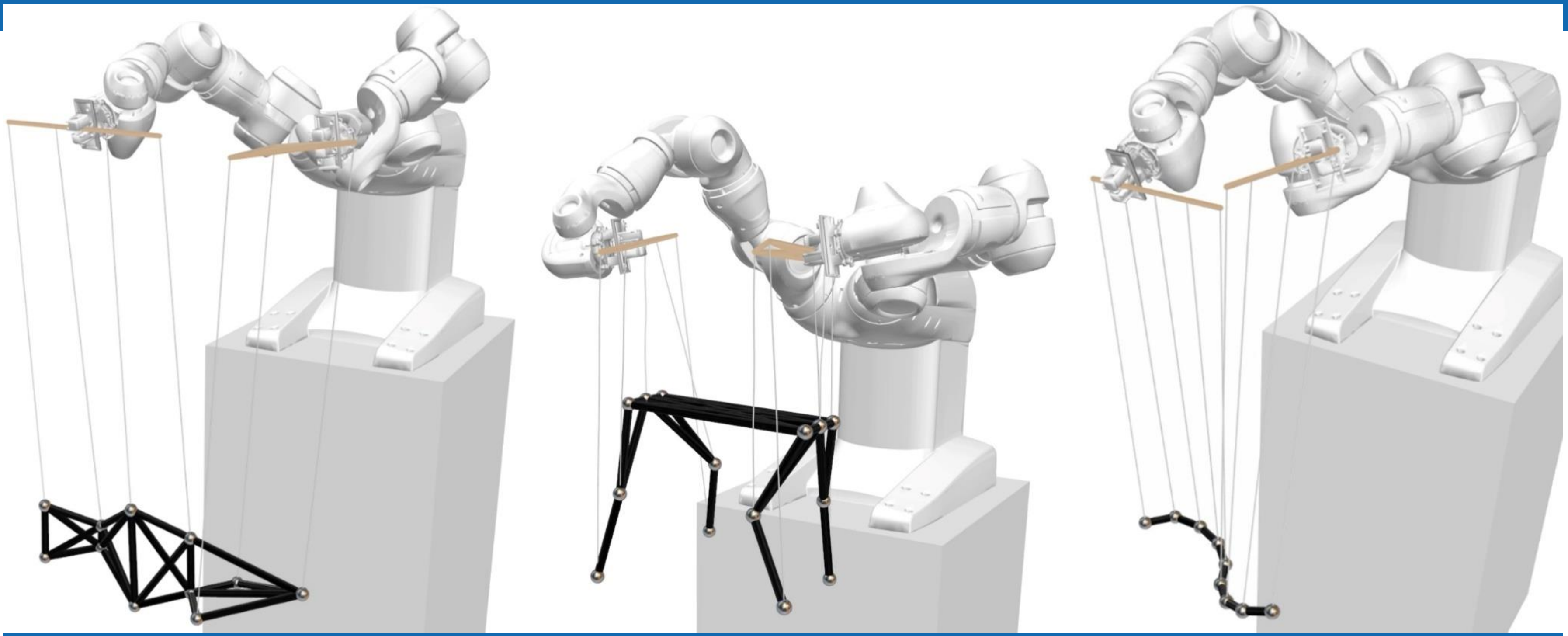
ETH zürich



Puppet Master

Simon Zimmermann

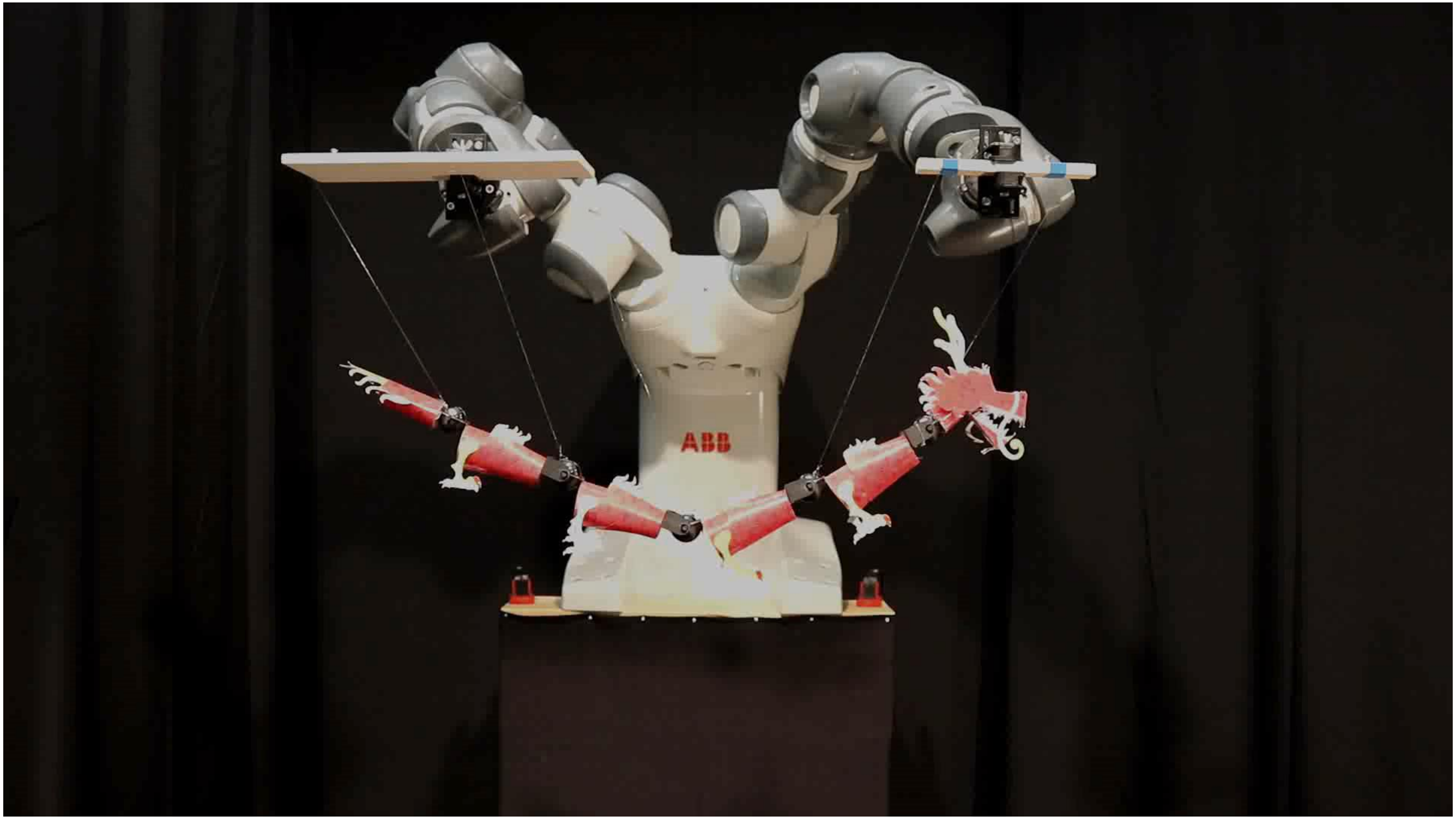
Dr. Roi Poranne

James Bern

Prof. Dr. Stelian Coros

CRL

First Example



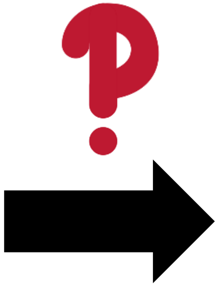
Introduction

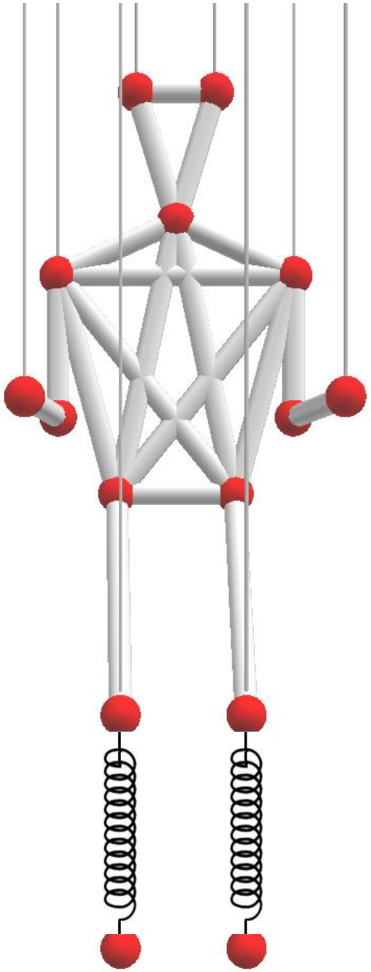
Newton's Method

Forward Simulation

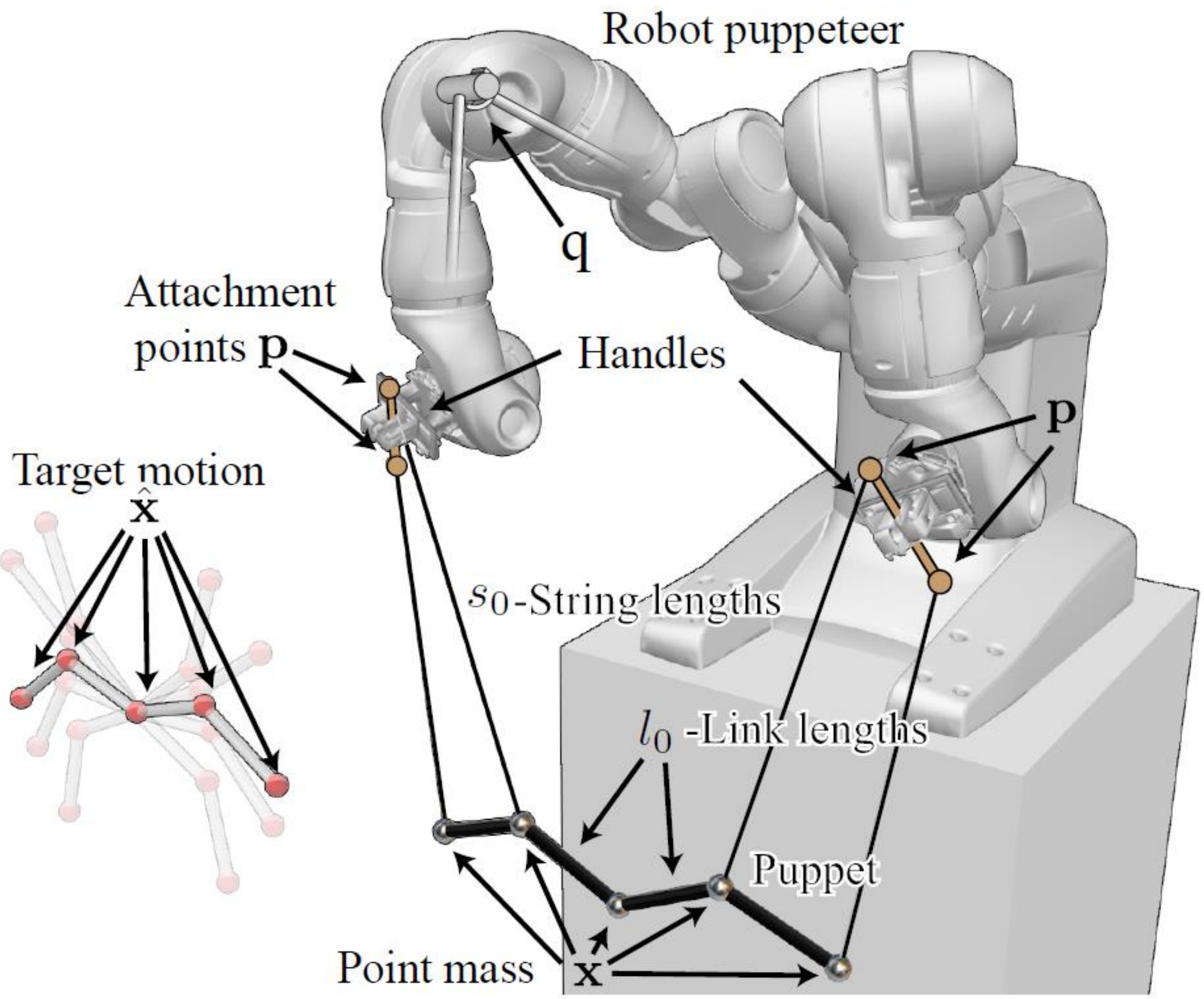
CRL

Modelling





System Overview



Dynamics – Forward Integration

$$\dot{x} = f(t, x), \qquad x(t_0) = x_0$$

Implicit:

$$x_{k+1} = x_k + hf(t_{k+1}, x_{k+1})$$

Explicit:

$$x_{k+1} = x_k + hf(t_k, x_k)$$

Dynamics – Forward Simulation

$$\boldsymbol{x}_i(\boldsymbol{p}_i) = \underset{\boldsymbol{x}_i}{\operatorname{argmin}}\, U_i(\boldsymbol{x}_i, \boldsymbol{p}_i)$$

where $$U_i(\boldsymbol{x}_i, \boldsymbol{p}_i) = \frac{h^2}{2} \ddot{\boldsymbol{x}}_i^T M \ddot{\boldsymbol{x}}_i + W(\boldsymbol{x}_i, \boldsymbol{p}_i) + \boldsymbol{g}^T M \boldsymbol{x}_i$$

$$\longrightarrow \quad \ddot{\boldsymbol{x}}_i \approx \frac{\boldsymbol{x}_i - 2\boldsymbol{x}_{i-1} + \boldsymbol{x}_{i-2}}{h^2}$$

$$\longrightarrow \quad W(\boldsymbol{x}_i, \boldsymbol{p}_i) = W_{puppet}(\boldsymbol{x}_i) + W_{string}(\boldsymbol{x}_i, \boldsymbol{p}_i)$$

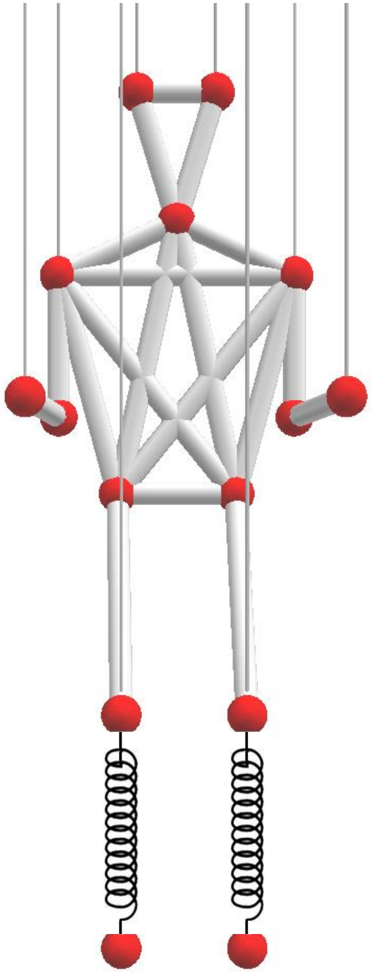
Dynamics – Forward Simulation

$$W(\boldsymbol{x}_i, \boldsymbol{p}_i) = W_{puppet}(\boldsymbol{x}_i) + W_{string}(\boldsymbol{x}_i, \boldsymbol{p}_i)$$

$$W_{puppet}(\boldsymbol{x}_i) = \sum_{(j_1,j_2)\in L} k_s \left(\left\| \boldsymbol{x}_i^{j_1} - \boldsymbol{x}_i^{j_2} \right\| - l_0^{j_1,j_2} \right)^2$$

$$W_{string}(\boldsymbol{x}_i) = \sum_{(j_1,j_2)\in S} k_s \, \psi \left(\left\| \boldsymbol{x}_i^{j_1} - \boldsymbol{x}_i^{j_2} \right\| - s_0^{j_1,j_2} \right)$$

$$\psi(x) = \begin{cases} \frac{1}{2}x^2 + \frac{\epsilon}{2}x + \frac{\epsilon^2}{6} & x > 0 \\ \frac{1}{6\epsilon}x^3 + \frac{1}{2}x^2 + \frac{\epsilon}{2}x + \frac{\epsilon^2}{6} & x > -\epsilon \\ 0 & \text{otherwise} \end{cases}$$



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Control – Sensitivity Analysis

Objective: $\min_{\boldsymbol{p}} O(\boldsymbol{x}(\boldsymbol{p}), \boldsymbol{p}) \longrightarrow O(\boldsymbol{x}(\boldsymbol{p}), \boldsymbol{p}) = \|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2^2 + \|\ddot{\boldsymbol{p}}\|_2^2$

Gradient: $\frac{dO}{d\boldsymbol{p}} = \frac{\partial O}{\partial \boldsymbol{x}} \frac{d\boldsymbol{x}}{d\boldsymbol{p}} + \frac{\partial O}{\partial \boldsymbol{p}}$

$$\boxed{\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{p}) = \boldsymbol{0}} \longrightarrow \frac{d\boldsymbol{G}}{d\boldsymbol{p}} = \boldsymbol{0} = \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{x}} \frac{d\boldsymbol{x}}{d\boldsymbol{p}} + \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}}$$

$$\boxed{\boldsymbol{f} - \boldsymbol{ma} = \boldsymbol{0}}$$

$$\longrightarrow \frac{d\boldsymbol{x}}{d\boldsymbol{p}} = -\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{x}}\right)^{-1} \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} := \boldsymbol{S}$$

$$\nabla_{\boldsymbol{x}_i} U_i(\boldsymbol{x}_i) = \nabla_{\boldsymbol{x}_i} \left(\frac{h^2}{2} \ddot{\boldsymbol{x}}_i^T M \ddot{\boldsymbol{x}}_i + W(\boldsymbol{x}_i, \boldsymbol{p}_i) + \boldsymbol{g}^T M \boldsymbol{x}_i \right)$$

$$= M\ddot{\boldsymbol{x}}_i + \nabla_{\boldsymbol{x}_i} W(\boldsymbol{x}_i, \boldsymbol{p}_i) + M\boldsymbol{g} = M\ddot{\boldsymbol{x}}_i - \boldsymbol{F}(\boldsymbol{x}_i, \boldsymbol{p}_i) := \boldsymbol{G}$$

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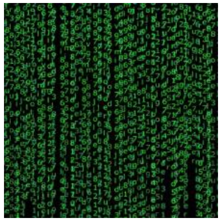
Second Order Sensitivity Analysis

Newton's Method: $$\frac{d^2 O}{d\boldsymbol{p}^2}\Delta\boldsymbol{p} = -\frac{dO}{d\boldsymbol{p}}$$

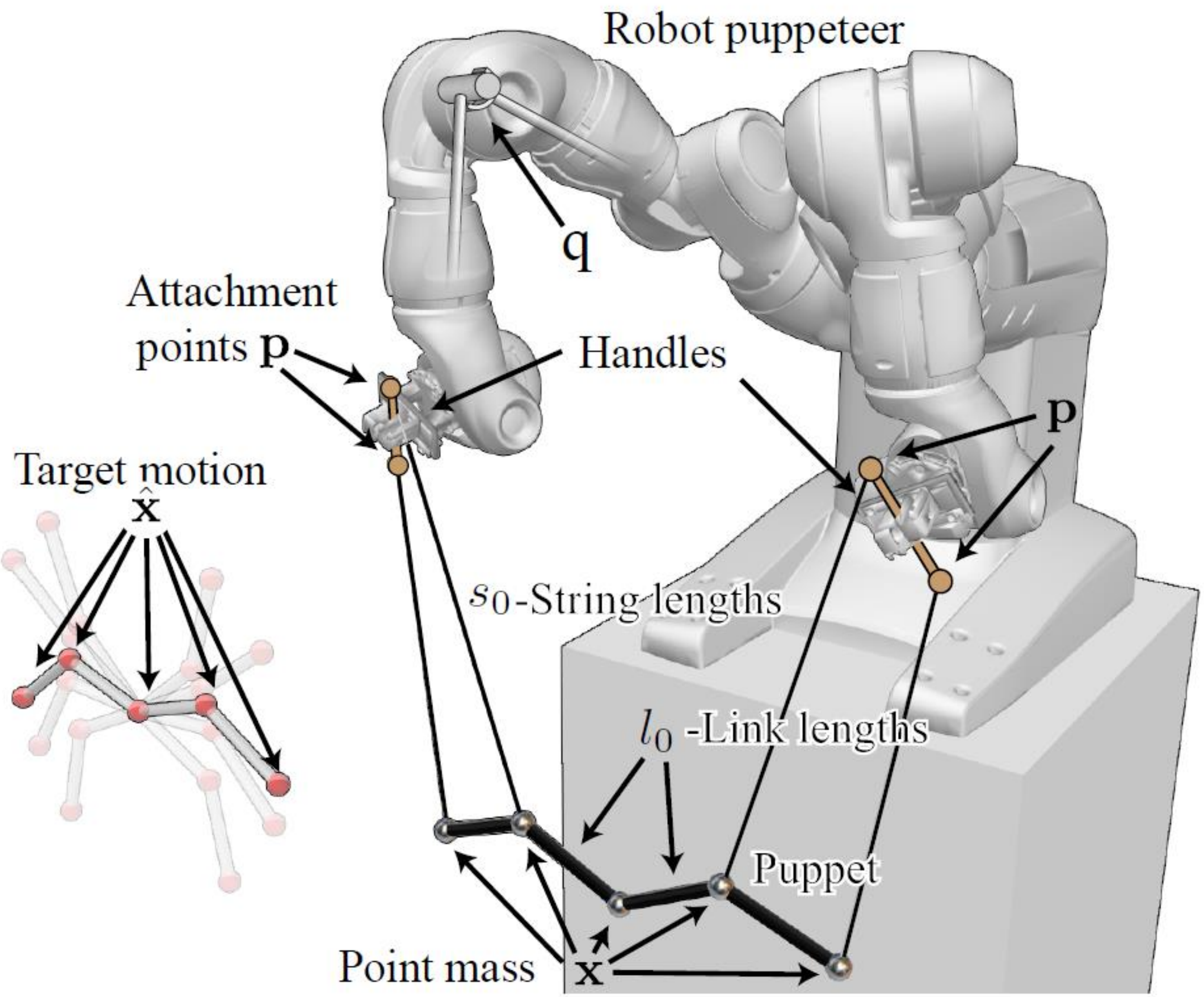
$$\frac{d^2 O}{d\boldsymbol{p}^2} = \frac{d}{d\boldsymbol{p}}\frac{dO}{d\boldsymbol{p}} = \frac{d}{d\boldsymbol{p}}\left(\frac{\partial O}{\partial \boldsymbol{x}}\boldsymbol{S} + \frac{\partial O}{\partial \boldsymbol{p}}\right)$$

$$\vdots$$



$$\frac{d^2 O}{d\boldsymbol{p}^2} = \frac{\partial O}{\partial \boldsymbol{x}}\left(\boldsymbol{S}^T\frac{\partial}{\partial \boldsymbol{x}}\boldsymbol{S} + \frac{\partial}{\partial \boldsymbol{p}}\boldsymbol{S}\right) + \boldsymbol{S}^T\left(\frac{\partial^2 O}{\partial \boldsymbol{x}^2}\boldsymbol{S} + 2\frac{\partial O}{\partial \boldsymbol{x}\partial \boldsymbol{p}}\right) + \frac{\partial^2 O}{\partial \boldsymbol{p}^2}$$

Joint Space

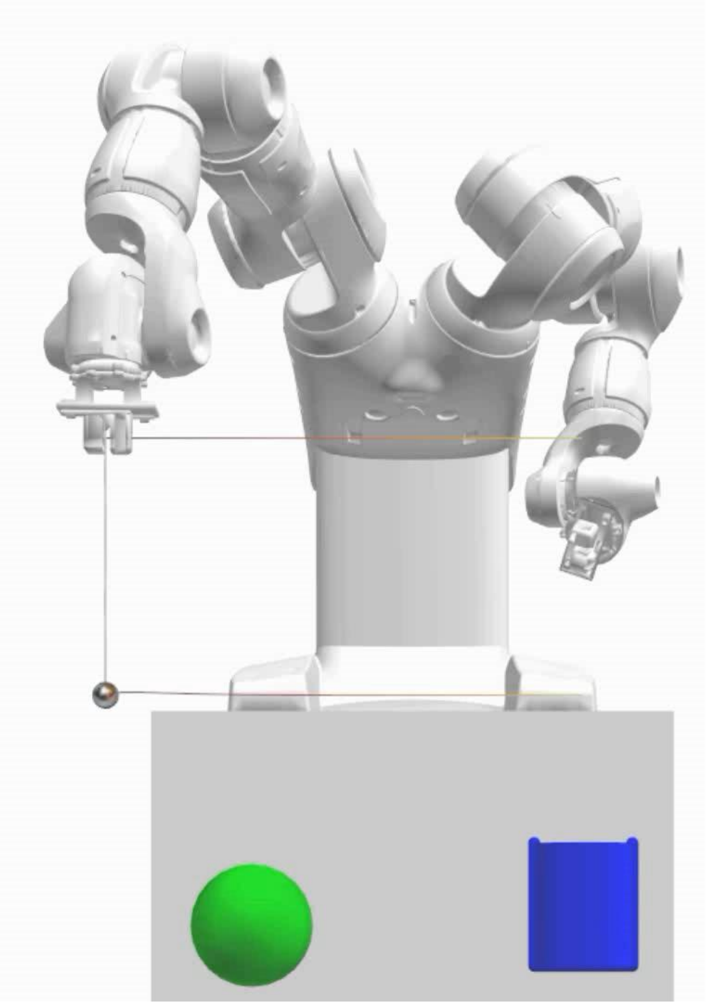


$$\min_{\boldsymbol{q}} O\big(\boldsymbol{x}(\boldsymbol{p}(\boldsymbol{q})), \boldsymbol{p}, \boldsymbol{q}\big)$$

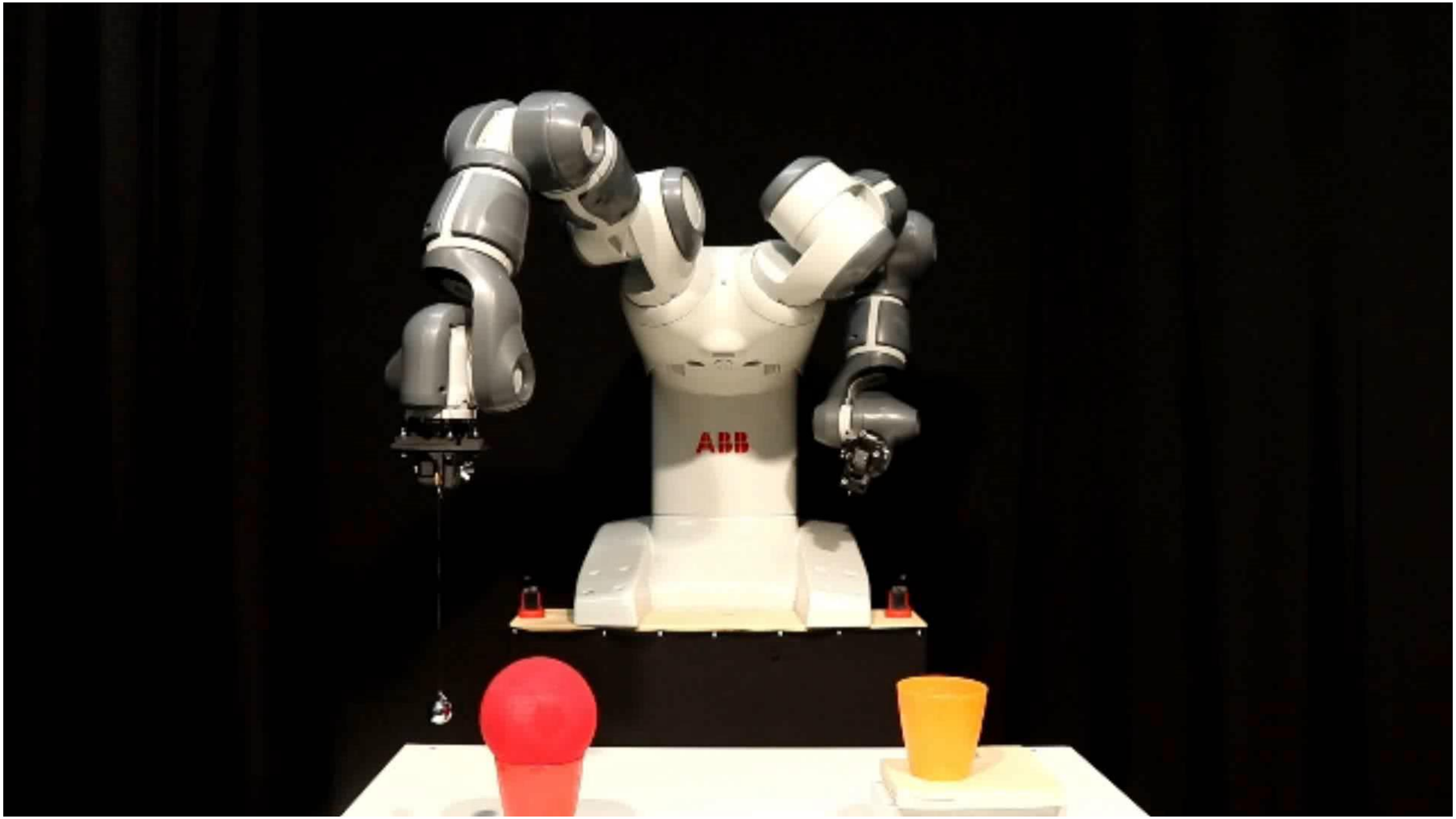
$$\frac{dO}{d\boldsymbol{q}} = \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{q}}^T \frac{dO}{d\boldsymbol{p}} + \frac{\partial O}{\partial \boldsymbol{q}}$$

Preliminary Experiments – Pendulum

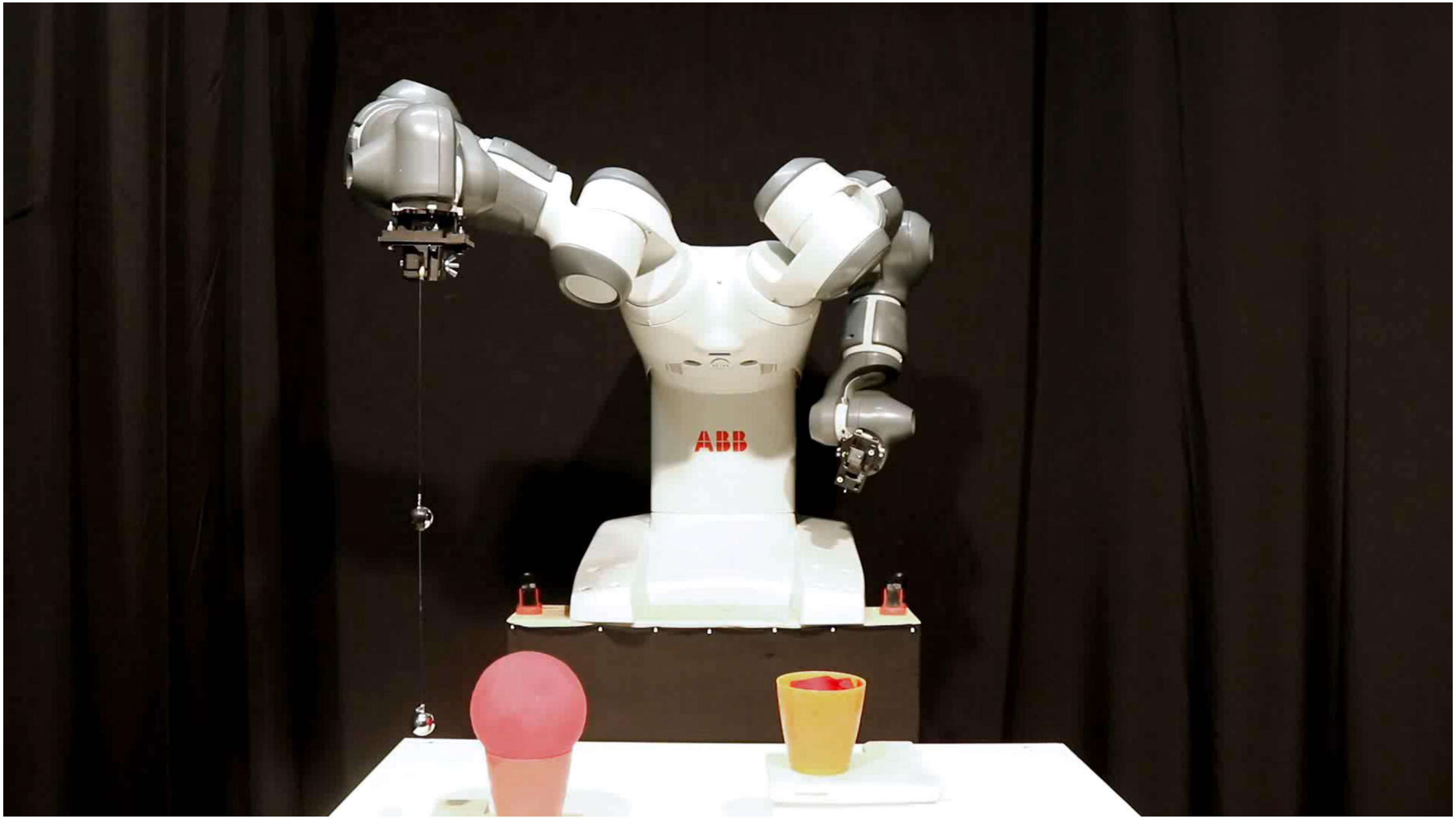
Task:
Avoid obstacle
and jump into cup
(robot hand constrained
to move along horizontal
axis)



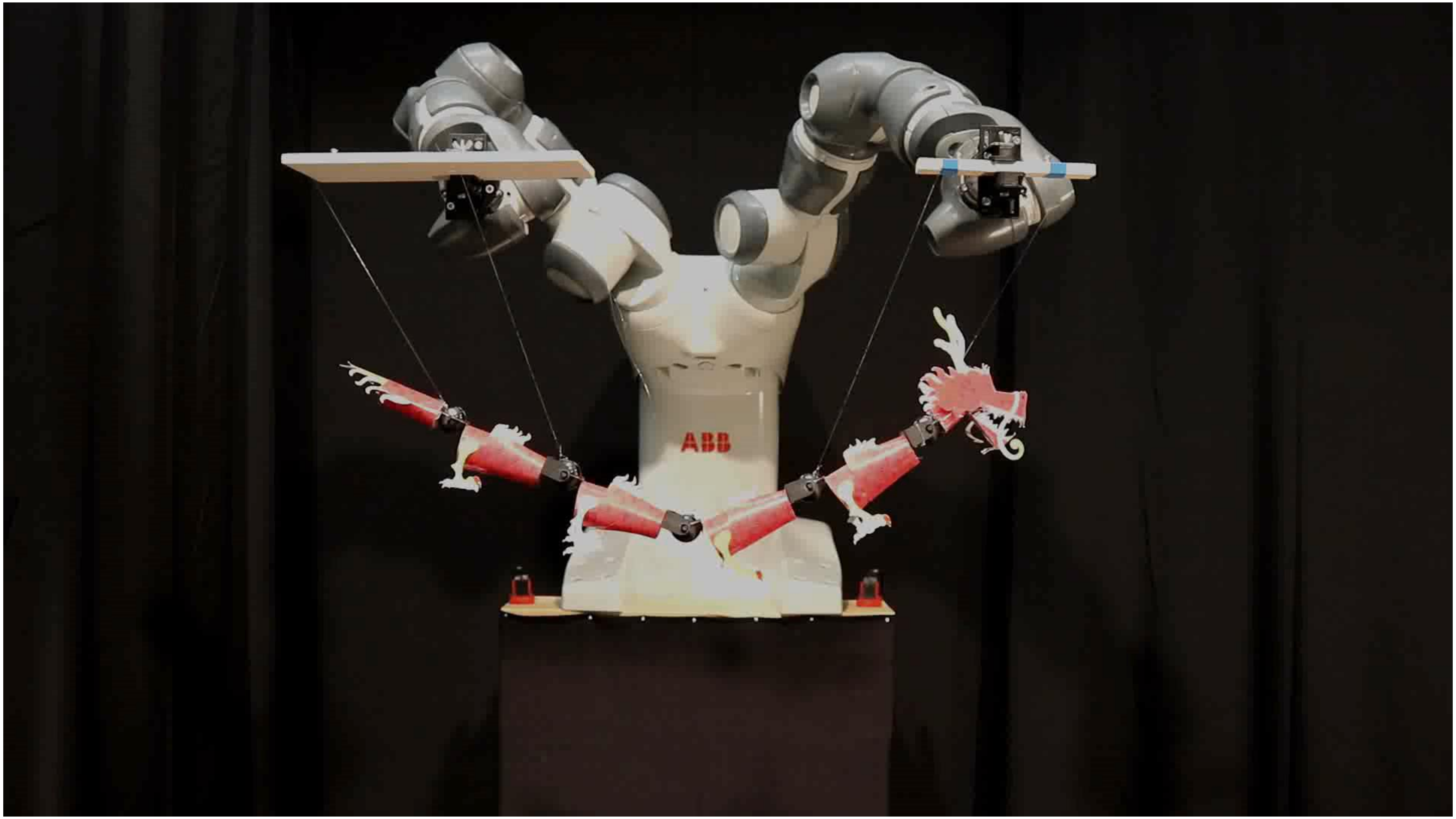
Preliminary Experiments – Pendulum



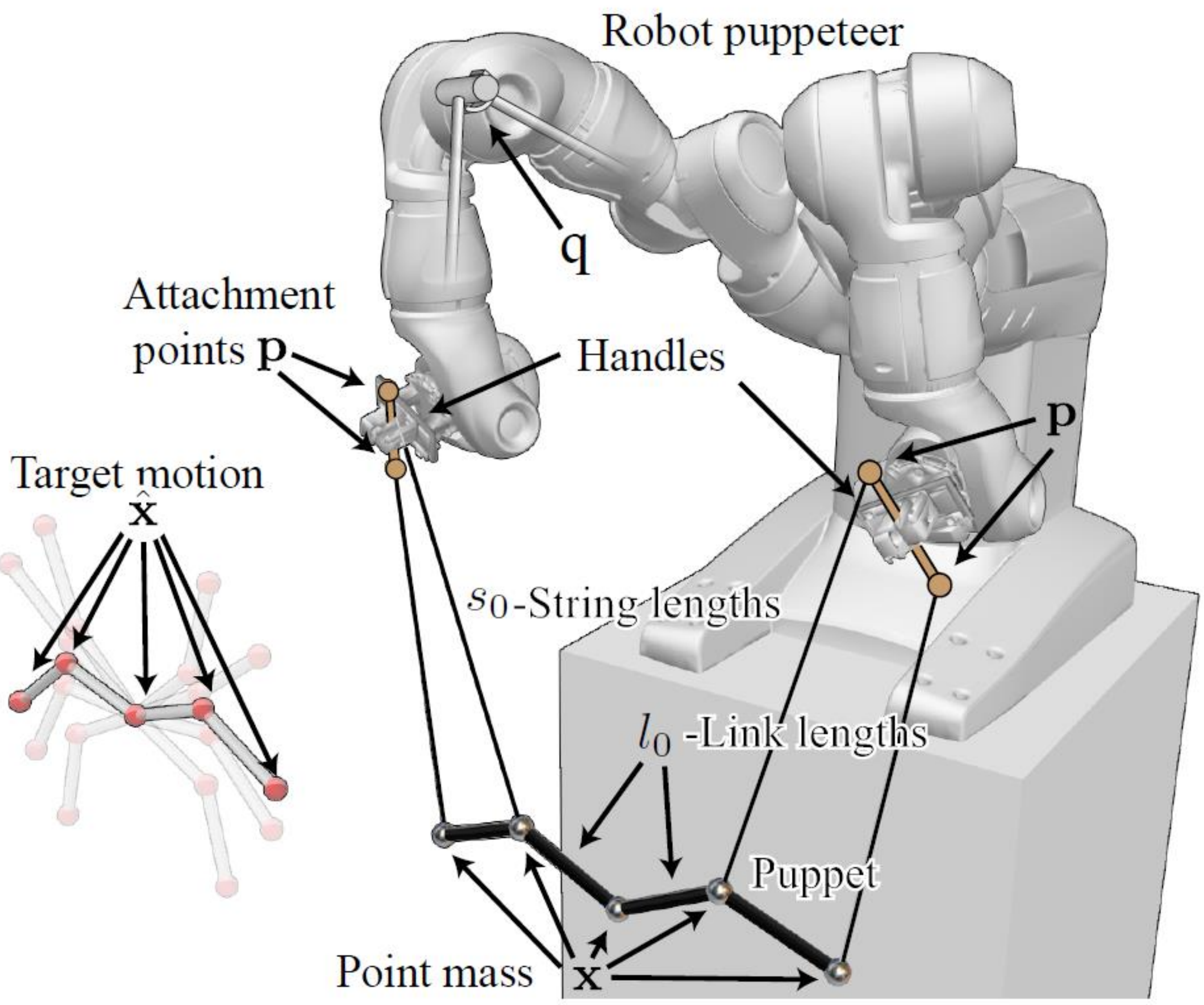
Preliminary Experiments – Double Pendulum



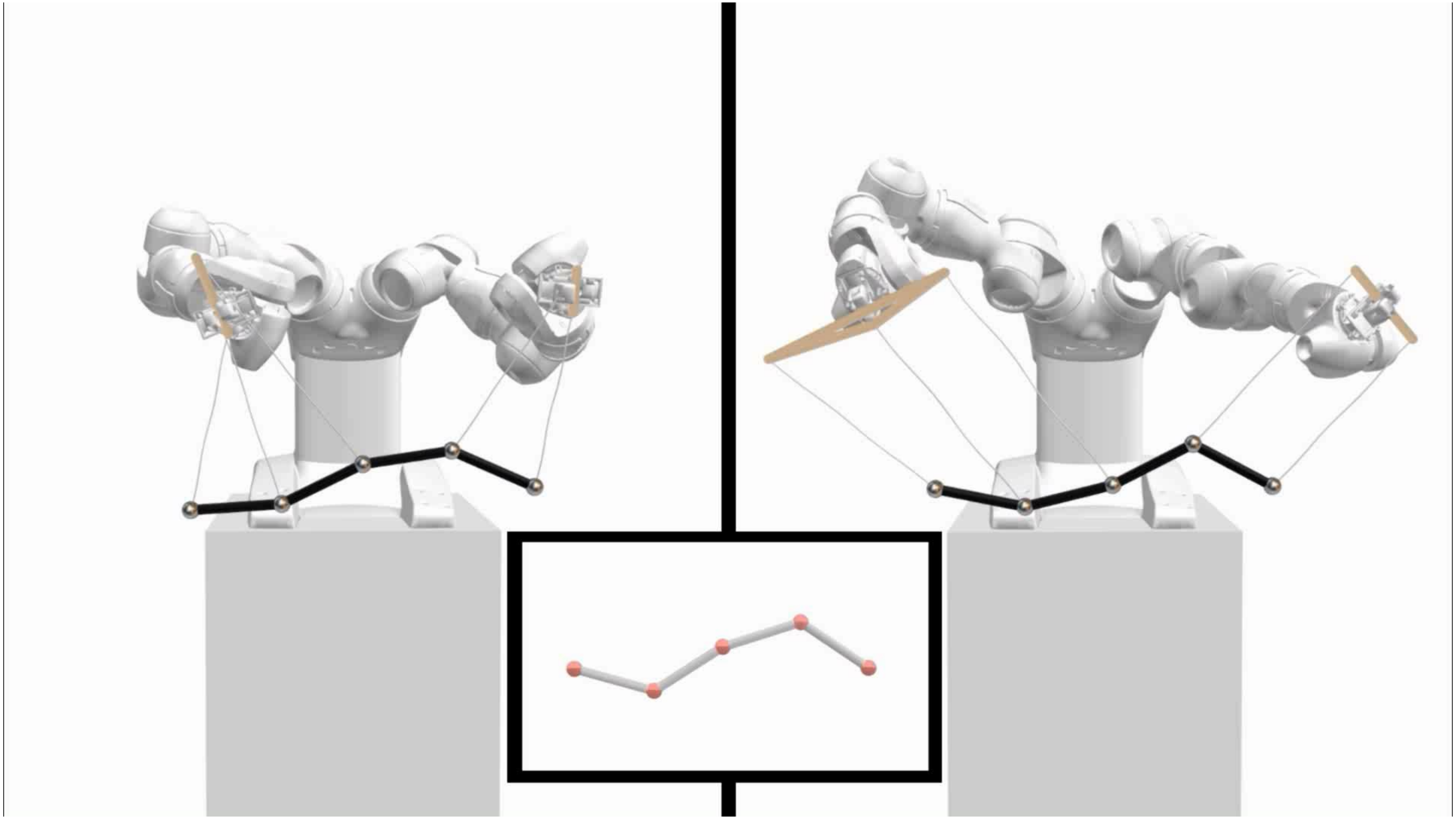
Physical Puppet: Chinese Dragon



Design Optimization



Design Optimization

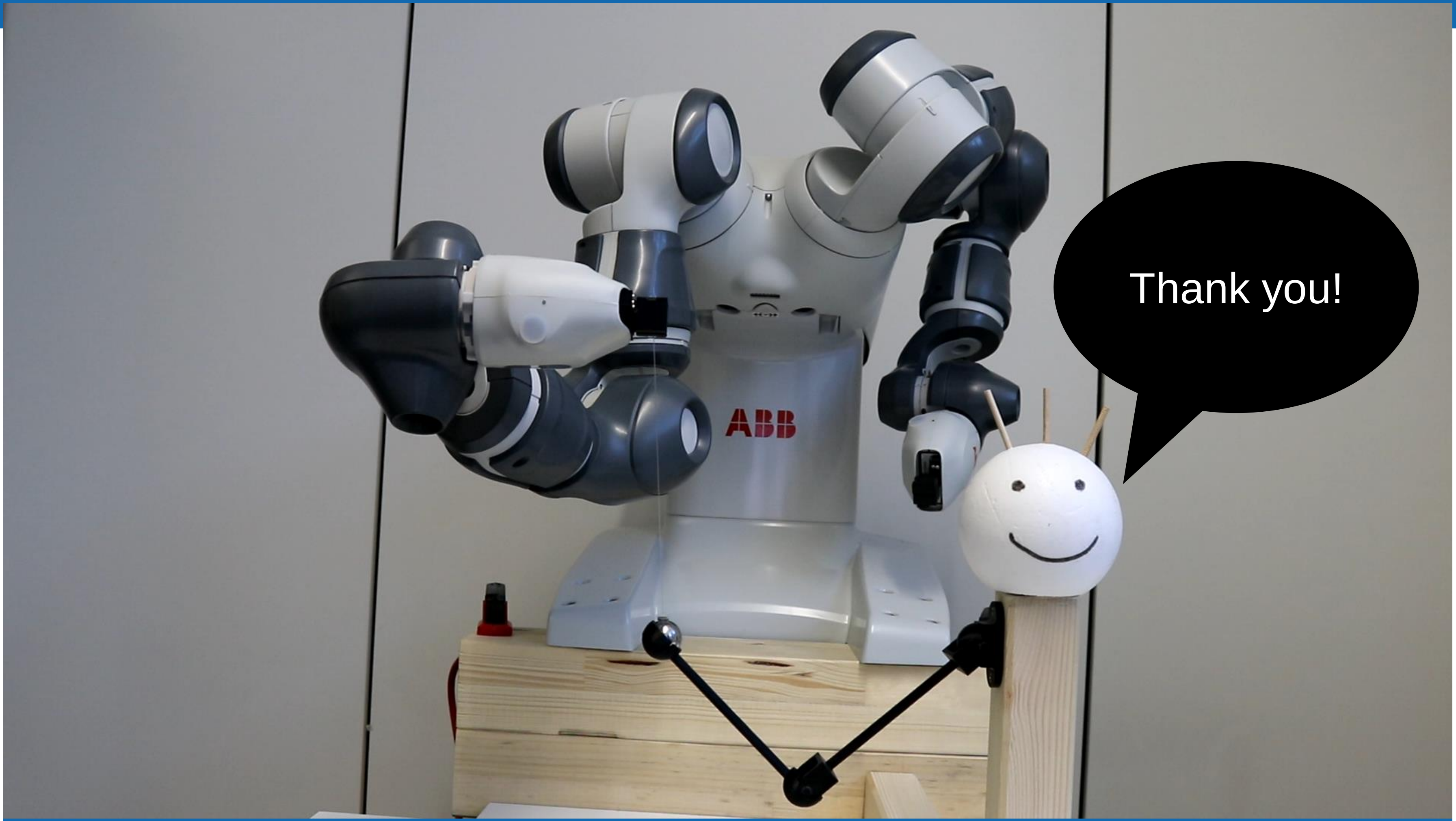


Simulation Results

Simulation Results

Physical Results

Physical Results


Thank you!
ABB